PHYS 301 Tutorial #2 Quiz Sol'ns

1. if $\vec{v} = \vec{\nabla} \times \vec{A}$, what is $\vec{\nabla} \times (\vec{A} + \vec{\nabla} f)$?

$$\vec{\nabla} \times (\vec{A} + \vec{\nabla} f) = \underbrace{\vec{\nabla} \times \vec{A}}_{= \vec{v}} + \underbrace{\vec{\nabla} \times (\vec{\nabla} f)}_{= 0 \ \forall f}$$

$$\boxed{\therefore \ \vec{\nabla} \times (\vec{A} + \vec{\nabla} f) = \vec{v}}$$

$\therefore$ if $\vec{v} = \vec{\nabla} \times \vec{A}$, then we can add $\vec{\nabla} f$ to $\vec{A}$ while maintaining the identity $\vec{v} = \vec{\nabla} \times \vec{A}$

2. If $W = \int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{l}$ is path independent and depends only on the end pts $\vec{a}$ & $\vec{b}$, then:

$$\oint \vec{F} \cdot d\vec{l} = 0$$

since, in this case, the starting & ending pts are the same.

By Stoke's theorem,

$$\int (\vec{\nabla} \times \vec{F}) \cdot d\vec{a} = \underbrace{\oint \vec{F} \cdot d\vec{l}}_{0 \text{ for conservative forces}}$$

$$\boxed{\therefore \vec{\nabla} \times \vec{F} = 0}$$